<u>Squeeze Theorem -</u>

1. Let g and h be the functions defined by $g(x) = -x^2 + 2x - 3$ and $h(x) = 2x + 1$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for all x, what is $\lim_{x \to -2} f(x)$?	2. Find $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

Intermediate Value Theorem (IVT)

x	-5	1	3	8	14
$f(x)$	7	40	21	75	-100

3. The table above gives selected values of a continuous function *f*. Use the table to answer the following questions.

a. On the interval $-5 \le x \le 1$, must there be value *r*, such that $f(r) = 30$? Explain.	b. On the interval $-5 \le x \le 14$, what is the minimum number of zeros?	c. On the interval $-5 \le x \le 14$, what is the fewest possible times $f(x) = 20$?

4. Consider the function $s(x) = 2x^2 + 4x$. Must there be some *a* between 0 and 1 for which $s(a) = 3$? Explain your reasoning.

5. Let *k* be the function defined by $k(x) = \begin{cases} x+2, & x \le 1 \\ x^2 + 4x - 2, & x > 1 \end{cases}$. Must there exist some *t* in the interval $(0,3)$ for which $k(t) = 4$? Explain why or why not.

Extreme Value Theorem:

If a function f is continuous over the interval $[a, b]$, then f has at least one minimum value and at least one maximum value on $[a, b]$.

A table is shown with selected function values for the twice differentiable function k. Read each of the explanations that follow and decide whether the Intermediate Value Theorem or the Extreme Value Theorem applies. Fill in the blanks with **IVT** or **EVT**, as appropriate.

x	1	2	3	4	5	6	7
$k(x)$	5	2	-4	-1	3	2	0

1. Since k is differentiable, it is also continuous. Since $k(6) = 2$ and $k(7) = 0$, and since 1 is between 2 and 0, it follows by the $\boxed{}$ that $k(c) = 1$ for some c between 6 and 7.

2. There must be a minimum value for k at some r in $[1, 7]$, because k is differentiable and therefore also continuous. Hence the $\boxed{}$ applies.

3. There must be some value a in $(2, 6)$ for which $f'(a) = 0$, because k is continuous and therefore k has an absolute maximum on $[0, 6]$ by the $\boxed{}$.